

## Kepler, Newton, and Orbital Motion - Answers and Advice

1. Just like in class:

$$(a) a^3 / P^2 = (1 \text{ AU})^3 / (1 \text{ year})^2 = 1 \text{ AU}^3 / \text{year}^2 \text{ or just } 1.$$

$$(b) a^3 / P^2 = (9.539 \text{ AU})^3 / (29.46 \text{ year})^2 = 1 \text{ AU}^3 / \text{year}^2$$

2. For objects orbiting the Sun we know that  $1 = a^3 / P^2$  so for Pluto we have

$$1 = (39.54)^3 / P^2$$

$$P^2 = (39.54)^3 = 61817 \text{ years}^2$$

$$P = 249 \text{ years}$$

3. For objects orbiting the Sun, we know  $1 = a^3 / P^2$  so for Comet Halley we have

$$1 = a^3 / (76)^2$$

$$(76)^2 = a^3$$

$$a^3 = (76)^2 = 5776 \text{ AU}^3$$

$$a = 17.9 \text{ AU}$$

4. This is similar to the previous ones, but it doesn't involve something orbiting the Sun.

$$\text{If } a \text{ is in AU and } P \text{ is in years then (mass in "suns")} = \frac{a^3}{P^2}$$

$$\therefore (\text{mass in "suns"}) = \frac{a^3}{P^2} = \frac{(20 \text{ AU})^3}{(50 \text{ years})^2} = 3.2 \text{ solar masses}$$

5. Use the same method employed in the previous solution.

$$\therefore (\text{mass in "suns"}) = \frac{a^3}{P^2} = \frac{(15 \text{ AU})^3}{(41 \text{ years})^2} = 2.0 \text{ solar masses}$$

6. Just use Kepler's third law as modified by Isaac Newton:

$$\frac{a^3}{p^2} = \frac{G}{4\pi^2} (m_1 + m_2) \Rightarrow \frac{(1.05 \times 10^9 \text{ m})^3}{(6.18 \times 10^5 \text{ sec})^2} = \frac{6.67 \times 10^{-11}}{4(3.14159)^2} (m_J + m_G)$$

Now rearrange and solve for  $(m_J + m_G)$ .

$$(m_J + m_G) = \frac{4(3.14159)^2}{6.67 \times 10^{-11}} \times \frac{(1.05 \times 10^9 \text{ m})^3}{(6.18 \times 10^5 \text{ sec})^2} = 1.90 \times 10^{27} \text{ kg}$$

This is essentially the mass of Jupiter alone. (Why?)

7. The solution is essentially the same as for the previous problem.

$$\frac{a^3}{p^2} = \frac{G}{4\pi^2} (m_1 + m_2) \Rightarrow \frac{(1.90 \times 10^7 \text{ m})^3}{(5.52 \times 10^5 \text{ sec})^2} = \frac{6.67 \times 10^{-11}}{4(3.14159)^2} (m_P + m_C)$$

$$(m_P + m_C) = \frac{4(3.14159)^2}{6.67 \times 10^{-11}} \times \frac{(1.90 \times 10^7 \text{ m})^3}{(5.52 \times 10^5 \text{ sec})^2} = 1.33 \times 10^{22} \text{ kg}$$

8.

<b>Moon</b>	<b>a</b>	<b>P</b>	<b><math>a^3/p^2</math> (<math>\text{m}^3 / \text{sec}^2</math>)</b>
Europa	409,300 miles = $6.587 \times 10^8 \text{ m}$	3.55 days = 306,700 sec	$3.04 \times 10^{15}$
Ganymede	653,000 miles = $1.051 \times 10^9 \text{ m}$	7.15 days = 617,760 sec	$3.04 \times 10^{15}$
Callisto	1,149,000 miles = $1.848 \times 10^9 \text{ m}$	16.69 days = 1,442,000 sec	$3.04 \times 10^{15}$